

Original Article

Generalization of total least-squares on example of unweighted and weighted 2D similarity transformation

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Abstract

In this contribution it is shown that the so-called “total least-squares estimate” (TLS) within an errors-in-variables (EIV) model can be identified as a special case of the method of least-squares within the nonlinear Gauss–Helmert model. In contrast to the EIV-model, the nonlinear GH-model does not impose any restrictions on the form of functional relationship between the quantities involved in the model. Even more complex EIV-models, which require specific approaches like “generalized total least-squares” (GTLS) or “structured total least-squares” (STLS), can be treated as nonlinear GH-models without any serious problems. The example of a similarity transformation of planar coordinates shows that the “total least-squares solution” can be obtained easily from a rigorous evaluation of the Gauss–Helmert model. In contrast to weighted TLS, weights can then be introduced without further limitations. Using two numerical examples taken from the literature, these solutions are compared with those obtained from certain specialized TLS approaches.

Keywords Method of least-squares - Total least-squares (TLS) - Structured total least-squares (STLS) - Weighted total least-squares (WTLS) - Gauss–Helmert model - Gauss–Markov model - Coordinate transformation (2D)