INVESTGATION OF AXES ERRORS OF TERRESTRIAL LASER SCANNERS

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ABSTRACT:

For polar measuring systems (including terrestrial laser scanners), that are able to perform measurements in two faces of the telescope, it is shown how the collimation error, the tilting axis error and the eccentricity of the collimation axis (line of sight) can be derived from over-determined measurements using a least squares adjustment. The axes errors are illustrated and the application of newly derived formulas demonstrated for the terrestrial laser scanner Zoller + Fröhlich Imager 5003.

1. INTRODUCTION

If a polar measuring system (e.g. tacheometer, terrestrial laser scanner) is affected by axes errors and axes eccentricities, the direction and angular measurements are falsified by the influence of these errors. Here, the following errors are considered:

- Tilting axis error *i*,
- Collimation error *c*,
- Eccentricity of the line of sight (collimation axis) *e*.

Is should be noted that these three errors do not model all possible error influences; further errors are listed in (Deumlich & Staiger 2002, pp. 205 et seq.), for example. But we can assume that the errors *i*, *c* and *e* are the largest errors affecting the direction and angular measurements.

When taking measurements with a tacheometer the influences of the mentioned errors can be eliminated by pointing to an object in the two faces of the telescope. If, for economical reasons, the measurements are only performed in one face of the telescope, the axes errors and the eccentricity of the collimation axis have to be determined in advance. Using these calibration values, the corrected directions and angles can be computed,

Even if an object can be scanned in two faces by a laser scanner, it is impossible to eliminate the influences of the axes errors and the eccentricity of the collimation axis (line of sight). Since the point clouds derived from the measurements in Face 1 and Face 2 of the sensor do not exactly overlap, an exact matching of corresponding points is not possible. But there is the possibility to determine the errors before the scanning process using a suitable measurement procedure. The corrections determined can then be used to correct the results of a scanning process in one face of the sensor. Below it will be shown how the errors can be derived from an over-determined measurement configuration using a least squares adjustment.

2. POLAR MEASUREMENT SYSTEM AS A REALISATION OF A LOCAL COORDINATE SYSTEM

For the determination of the errors of the axes, directions and angles need to be referenced to the instrument only. An adjustment of the rotation axis of the instrument to the plumb direction (i.e. 'levelling of the sensor') is irrelevant in this context. If the instrument is equipped with inclination sensors ('level sensors') they have to be switched off during the determination of the axes errors.

In this paper a polar measuring system is considered as a realisation of a local coordinate system with an arbitrary orientation. This coordinate system (see Figure 1) is called the *instrument system*.

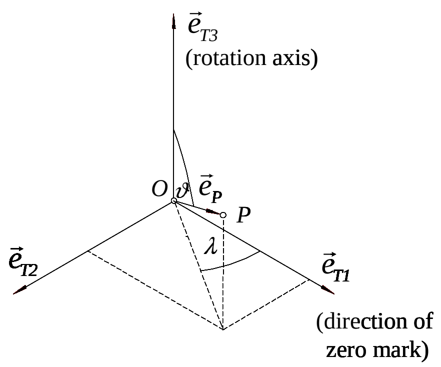

Figure 1: Instrument system

In order to describe the measurements with a polar measuring system whose rotation axis is not aligned with the plumb direction, the designations shown in Figure 2 are used for the axes and encoders.

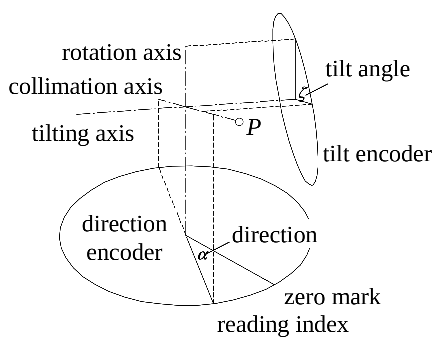

Figure 2: Axes and encoders of a polar measuring system

The goal of direction and angular measurements with a polar measuring system is to describe the direction to a target point *P* by the spherical coordinates λ und ϑ in the instrument system. The direction to the target point *P* is described by the unit vector $\vec{e}_P$, see Figure 1.

Since the realisation of the abstract instrument system in a polar measuring system can never be perfect due to the inevitable errors of the manufacture of the instrument, we obtain the direction α and the tilt angle ζ instead of the desired spherical coordinates λ and ϑ when sighting to a target point *P* (see Figure 2). The direction α and the tilt angle ζ are defined as follows (Stahlberg 1997):

– α is an angle between two straight lines. The first straight line intersects the centre and the zero mark of the direction encoder, the second line intersects the centre of the direction encoder and the reading index. The second straight line is normal to the tilting axis.
– ζ is an angle between two planes. The first plane is defined by the tilting axis and the rotation axis. The second plane is formed by the tilting axis and the collimation axis (line of sight). Thus, ζ is an angle in a plane normal to the tilting axis.

From a multitude of mechanical inadequacies of the realisation of the instrument system (Deumlich & Staiger 2002, pp. 205 et seq.) that affect directions and angular measurements we select three errors for further discussion. These are the collimation error, the tilting axis error and the eccentricity of the collimation axis. The correction *f*, that needs to be added to a measured direction, and the associated tilt angle ζ can be obtained from measurements with a tacheometer in both faces of the telescope from

$$f = \frac{\alpha_{II} - 200\,\text{gon} - \alpha_I}{2} \quad \text{and} \qquad (1)$$

$$\zeta = \frac{\zeta_I + 400\,\text{gon} - \zeta_{II}}{2} \quad , \qquad (2)$$

where: α_I, α_{II} = directions in Faces 1 and 2 of the telescope
ζ_I, ζ_{II} = tilt angles in Faces 1 and 2 of the telescope

3. AXES AND AXES ERRORS

The common determination of the collimation error, the tilting axis error and the eccentricity of the collimation axis of a laser scanner with tacheometric measuring principle is shown for the instrument Zoller + Fröhlich Imager 5003. For laser scanners with a tacheometric measuring principle, the notation of the axes of a tacheometer can be used as follows:

– Rotation axis: Axis of rotation of the top part of the instrument during a scanning process.
– Tilting axis: Axis of rotation of the deflection mirror during a scanning process.
– Collimation axis: Assuming a conical propagation of the laser beam, the “collimation axis” (line of sight) is the straight line from the point the laser beam is deflected in the direction of an object (point Z) to the centre of the base of the cone (point Z').

The axes of the instrument are depicted in Figure 3 and Figure 4.

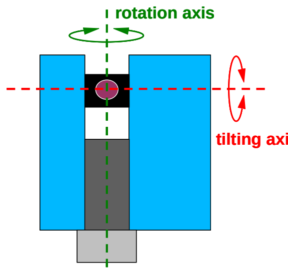

Figure 3: Rotation and tilting axes of Z + F Imager 5003

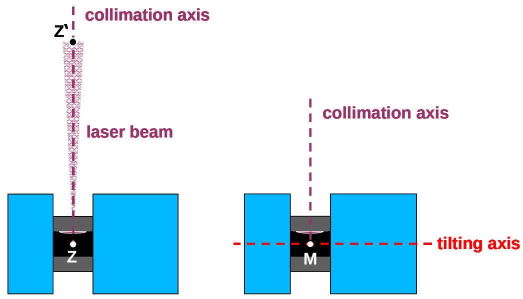

Figure 4: Collimation axis of Z + F Imager 5003

For an error free instrument the axes must fulfil the following conditions:

– Tilting axis is normal to the rotation axis;
– Collimation axis is normal to the tilting axis;
– Collimation axis intersects the rotation axis.

A tilting axis error is present if the tilting axis is not normal to the rotation axis. A collimation error exists if the axis of collimation (line of sight) is not normal to the tilting axis. If the collimation axis does not intersect the rotation axis an eccentricity of the collimation axis is present. These errors can be defined as follows:

– Tilting axis error *i*: Angle between the tilting axis of the deflection mirror and the normal to the rotation axis measured in the plane defined by the rotation axis and the tilting axis.
– Collimation error *c*: Angle between the collimation axis and the normal to the tilting axis measured in the plane formed by the tilting axis and the collimation axis.
– Eccentricity of the collimation axis *e*: Radius of the circle around the point M of the rotation axis that is described by the collimation axis (as a tangent to the circle) when rotating the top part of the instrument.

The axes errors and the eccentricity of the collimation axis are illustrated in Figure 5 and Figure 6.

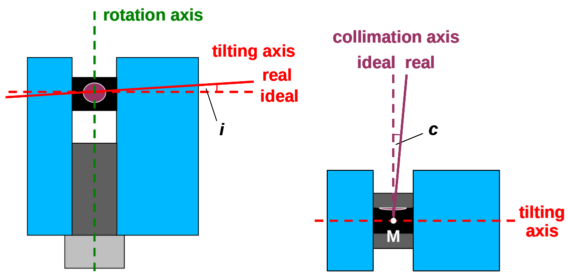

Figure 5: Tilting axis error and collimation error

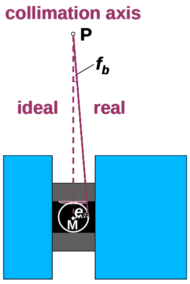

Figure 6: Eccentricity of the collimation axis

4. DETERMINATION OF AXES ERRORS

The correction f_a of directions for the effects of the axis errors *c* and *i*, as a function of the tilt angle ζ, can be described by (Stahlberg 1997):

$$f_a = \arctan\left(\frac{\cos i \tan c}{\sin \zeta} + \frac{\sin i}{\tan \zeta}\right) \quad . \tag{3}$$

The correction f_b (in radian) of the directions for the effect of the eccentricity of the collimation axis amounts to

$$f_b = \frac{e}{s} \quad , \tag{4}$$

(Deumlich & Staiger, 2002, p. 212), where *s* is the distance to a target point.

Summing f_a and f_b yields the total correction

$$f = f_a + f_b = \arctan\left(\frac{\cos i \tan c}{\sin \zeta} + \frac{\sin i}{\tan \zeta}\right) + \frac{e}{s} \quad . \tag{5}$$

Since *f* and ζ can be determined with Equations (1) and (2) from measurements in both faces of the telescope, the above equation contains three unknown parameters, namely *c*, *i* and *e*. For a solution, at least three such equations are required. Thus, measurements to three targets in Faces 1 and 2 of the telescope are necessary to determine the unknown parameters. Choosing more than three targets provides an over-determined configuration, which allows determining the axes errors and the eccentricity with a least squares adjustment.

4.1 Non-Linear Functional Model

From the measurements of directions and tilt angles to $n > 3$ targets in Faces 1 and 2 the following non-linear equations can be derived on the basis of Equation (5):

$$\begin{aligned} f_1 &= \arctan\left(\frac{\cos i \tan c}{\sin \zeta_1} + \frac{\sin i}{\tan \zeta_1}\right) + \frac{e}{s_1} \\ &\vdots \\ f_n &= \arctan\left(\frac{\cos i \tan c}{\sin \zeta_n} + \frac{\sin i}{\tan \zeta_n}\right) + \frac{e}{s_n} \end{aligned} \quad . \tag{6}$$

In order to solve for the unknown parameters with the help of an adjustment of the observation equations with one observation per observation equation, the tilt angles and their residuals as well as the distances and their residuals are regarded as additional unknown parameters, e.g. see (Koch 2000, pp. 88 et seq.), thus

$$\begin{matrix} \hat{\zeta}_1 = \zeta_1 + v_{\zeta_1} & & \hat{s}_1 = s_1 + v_{s_1} \\ \vdots & \text{and} & \vdots \\ \hat{\zeta}_n = \zeta_n + v_{\zeta_n} & & \hat{s}_n = s_n + v_{s_n} \end{matrix} \quad . \tag{7}$$

Considering the Equations (7) as additional observation equations, we obtain a total of 3*n* observation equations

$$\begin{aligned} f_1 + v_{f_1} &= \arctan\left(\frac{\cos i \tan c}{\sin \hat{\zeta}_1} + \frac{\sin i}{\tan \hat{\zeta}_1}\right) + \frac{e}{\hat{s}_1} \\ &\vdots \\ f_n + v_{f_n} &= \arctan\left(\frac{\cos i \tan c}{\sin \hat{\zeta}_n} + \frac{\sin i}{\tan \hat{\zeta}_n}\right) + \frac{e}{\hat{s}_n} \\ \zeta_1 + v_{\zeta_1} &= \hat{\zeta}_1 \\ &\vdots \\ \zeta_n + v_{\zeta_n} &= \hat{\zeta}_n \\ s_1 + v_{s_1} &= \hat{s}_1 \\ &\vdots \\ s_n + v_{s_n} &= \hat{s}_n \end{aligned} \tag{8}$$

for the determination of the unknown parameters *i*, *c* and *e* as well as $\zeta_1, \ldots, \zeta_n$ and $s_1, \ldots, s_n$.

This non-linear adjustment problem can be solved by linearisation and iterative computation using appropriate starting values c^0, i^0, e^0 and ζ_i^0, s_i^0 for the unknown parameters. With the following vectors of unknowns

$$\mathbf{x}_1 = \left[\Delta c \quad \Delta i \quad \Delta e\right]^{\mathrm{T}} \quad , \tag{9}$$

$$\mathbf{x}_2 = \left[\Delta \zeta_1 \quad \cdots \quad \Delta \zeta_n\right]^{\mathrm{T}} \quad , \tag{10}$$

$$\mathbf{x}_3 = \left[\Delta s_1 \quad \cdots \quad \Delta s_n\right]^{\mathrm{T}} \tag{11}$$

we receive the design matrices $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$ for the first *n* observation equations

$$\mathbf{A}_1 = \begin{bmatrix} \frac{\partial f_1}{\partial c} & \frac{\partial f_1}{\partial i} & \frac{\partial f_1}{\partial e} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial c} & \frac{\partial f_n}{\partial i} & \frac{\partial f_n}{\partial e} \end{bmatrix} \quad , \tag{12}$$

$$\mathbf{A}_2 = \begin{bmatrix} \frac{\partial f_1}{\partial \zeta_1} & \cdots & \frac{\partial f_1}{\partial \zeta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial \zeta_1} & \cdots & \frac{\partial f_n}{\partial \zeta_n} \end{bmatrix} , \tag{13}$$

$$\mathbf{A}_3 = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial s_1} & \cdots & \frac{\partial f_n}{\partial s_n} \end{bmatrix} . \tag{14}$$

The linearised observation vectors (absolute terms = measured minus computed) are

$$\mathbf{l}_1 = \begin{bmatrix} \cdots\cdots\cdots \\ f_i - \left\{ \arctan\left(\frac{\cos i^0 \tan c^0}{\sin \zeta_i^0} + \frac{\sin i^0}{\tan \zeta_i^0} \right) + \frac{e^0}{s_i^0} \right\} \\ \cdots\cdots\cdots \end{bmatrix} , \tag{15}$$

$$\mathbf{l}_2 = \begin{bmatrix} \zeta_1 - \zeta_1^0 \\ \vdots \\ \zeta_n - \zeta_n^0 \end{bmatrix} , \tag{16}$$

$$\mathbf{l}_3 = \begin{bmatrix} s_1 - s_1^0 \\ \vdots \\ s_n - s_n^0 \end{bmatrix} . \tag{17}$$

Together with the weight matrices $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$ of the observations f_i, ζ_i, s_i and the variance factor σ_0^2 we get the linear model as

$$\underbrace{\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 \\ \mathbf{0} & \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{l}_1 + \mathbf{v}_{l_1} \\ \mathbf{l}_2 + \mathbf{v}_{l_2} \\ \mathbf{l}_3 + \mathbf{v}_{l_3} \end{bmatrix} \quad \text{with} \tag{18}$$

$$D\left(\begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \\ \mathbf{l}_3 \end{bmatrix} \middle| \sigma_0^2 \right) = \sigma_0^2 \begin{bmatrix} \mathbf{P}_1^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_2^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P}_3^{-1} \end{bmatrix} , \tag{19}$$

where $\mathbf{E}$ is the unit matrix with dimension $(n \times n)$. The estimated values for the unknowns are obtained by iterative solution of the normal equations

$$\underbrace{\begin{bmatrix} \mathbf{A}_1^{\mathrm{T}}\mathbf{P}_1\mathbf{A}_1 & \mathbf{A}_1^{\mathrm{T}}\mathbf{P}_1\mathbf{A}_2 & \mathbf{A}_1^{\mathrm{T}}\mathbf{P}_1\mathbf{A}_3 \\ \mathbf{A}_2^{\mathrm{T}}\mathbf{P}_1\mathbf{A}_1 & \mathbf{A}_2^{\mathrm{T}}\mathbf{P}_1\mathbf{A}_2 + \mathbf{P}_2 & \mathbf{A}_2^{\mathrm{T}}\mathbf{P}_1\mathbf{A}_3 \\ \mathbf{A}_3^{\mathrm{T}}\mathbf{P}_1\mathbf{A}_1 & \mathbf{A}_3^{\mathrm{T}}\mathbf{P}_1\mathbf{A}_2 & \mathbf{A}_3^{\mathrm{T}}\mathbf{P}_1\mathbf{A}_3 + \mathbf{P}_3 \end{bmatrix}}_{\mathbf{N}} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1^{\mathrm{T}}\mathbf{P}_1\mathbf{l}_1 \\ \mathbf{A}_2^{\mathrm{T}}\mathbf{P}_1\mathbf{l}_1 + \mathbf{P}_2\mathbf{l}_2 \\ \mathbf{A}_3^{\mathrm{T}}\mathbf{P}_1\mathbf{l}_1 + \mathbf{P}_3\mathbf{l}_3 \end{bmatrix} . \tag{20}$$

The (a posteriori) standard deviation of the adjusted quantities $\hat{c}$, $\hat{i}$ and $\hat{e}$ as well as $\hat{\zeta}_1$, …, $\hat{\zeta}_n$ and $\hat{s}_1$, …, $\hat{s}_n$ can be computed with the help of the cofactor matrix of the unknowns.

$$\mathbf{Q}_{\mathrm{xx}} = \mathbf{N}^{-1} \tag{21}$$

and the estimated value of the variance factor $\hat{\sigma}_0^2$.

Numerical investigations with this approach have shown that the partial redundancies (redundancy numbers) of the observations ζ_1, …, ζ_n and s_1, …, s_n are so small compared with those of the observations f_1, …, f_n, that the observations ζ_i and s_i can be introduced as fixed parameters into the adjustment without significantly changing the adjusted values $\hat{c}$, $\hat{i}$, $\hat{e}$ and their standard deviations. In consequence a functional model is presented in the following section where the values ζ_i and s_i are introduced as fixed parameters.

4.2 Linear Functional Model

Introducing the tilt angles ζ_i and the distances s_i as fixed parameters and using the approximation

$$\tan f_a \approx f_a \quad , \tag{22}$$

under the assumption of small angles, and introducing the substitutions

$$a = \cos i \tan c \quad \text{and} \quad b = \sin i \tag{23}$$

we obtain the *linear* functional model

$$\begin{aligned} f_1 &= \frac{1}{\sin \zeta_1} a + \frac{1}{\tan \zeta_1} b + \frac{1}{s_1} e \\ &\vdots \\ f_n &= \frac{1}{\sin \zeta_n} a + \frac{1}{\tan \zeta_n} b + \frac{1}{s_n} e \end{aligned} \tag{24}$$

and the resulting observation equations

$$\begin{aligned} f_1 + v_{f_1} &= \frac{1}{\sin \zeta_1} a + \frac{1}{\tan \zeta_1} b + \frac{1}{s_1} e \\ &\vdots \\ f_n + v_{f_n} &= \frac{1}{\sin \zeta_n} a + \frac{1}{\tan \zeta_n} b + \frac{1}{s_n} e \end{aligned} . \tag{25}$$

With the vector of unknowns

$$\mathbf{x} = \begin{bmatrix} a & b & e \end{bmatrix}^{\mathrm{T}} \tag{26}$$

we get the design matrix

$$\mathbf{A} = \begin{bmatrix} \frac{1}{\sin\zeta_1} & \frac{1}{\tan\zeta_1} & \frac{1}{s_1} \\ \vdots & \vdots & \vdots \\ \frac{1}{\sin\zeta_n} & \frac{1}{\tan\zeta_n} & \frac{1}{s_n} \end{bmatrix} \quad . \tag{27}$$

The observation vector is

$$\mathbf{l} = [\, f_1 \quad \cdots \quad f_n \,]^{\mathrm{T}} \quad . \tag{28}$$

With the weight matrix **P** of the observations f_i and the variance factor σ_0^2, the linear model becomes

$$\mathbf{Ax} = \mathbf{l} \;\text{ with }\; D(\mathbf{l}\,|\,\sigma_0^2) = \sigma_0^2\mathbf{P}^{-1} \quad . \tag{29}$$

The estimated values for the unknowns $\hat{a}$, $\hat{b}$ and $\hat{e}$ are obtained directly (without iteration) by solving the normal equations

$$\underbrace{\mathbf{A}^{\mathrm{T}}\mathbf{PA}}_{\mathbf{N}}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{Pl} \quad . \tag{30}$$

The estimated values for the tilting axis error $\hat{i}$ and the collimation error $\hat{c}$ are obtained from Equation (23)

$$\hat{i} = \arcsin\hat{b} \quad \text{and} \quad \hat{c} = \arctan\left(\frac{\hat{a}}{\cos\hat{i}}\right) \quad . \tag{31}$$

The (a posteriori) standard deviation of the adjusted quantities $\hat{a}$, $\hat{b}$ and $\hat{e}$ can be computed with the help of the cofactor matrix of the unknowns

$$\mathbf{Q}_{\mathrm{xx}} = \mathbf{N}^{-1} \tag{32}$$

and the estimated variance factor $\hat{\sigma}_0^2$. The standard deviations $\sigma_{\hat{i}}$ and $\sigma_{\hat{c}}$ can be computed from an application of the law of the propagation of variances and covariance to Equations (31).

4.3 Example

In order to determine the collimation error, the tilting axis error and the eccentricity of the collimation axis of the laser scanner Zoller + Fröhlich Imager 5003, six points targeted with wooden spheres (diameter 0.15 m) were scanned in two faces of the sensor, see Figure 7.

Figure 7: Measurement set-up

The results of the measurement were the Cartesian coordinates of the surface scans of the spheres in the scanner coordinate system. Using these values, the coordinates of the centre of each sphere were computed with the aid of a least squares adjustment. From the coordinates of the spheres' centres, the directions $\alpha_{i_{\mathrm{I}}}$, $\alpha_{i_{\mathrm{II}}}$, the tilt angles $\zeta_{i_{\mathrm{I}}}$, $\zeta_{i_{\mathrm{II}}}$ and the distances $s_{i_{\mathrm{I}}}$, $s_{i_{\mathrm{II}}}$ could be computed in Faces 1 and 2.

The tilt angles ζ_i and the distances s_i listed in Table 1 were computed as mean values from the measurements in Faces 1 and 2. The corrections f_i to the measured directions (see in Table 1) were obtained from

$$f_i = \frac{\alpha_{i_{\mathrm{II}}} - \alpha_{i_{\mathrm{I}}}}{2} \quad . \tag{33}$$

Point	ζ [gon]	f [gon]	s [m]
1	14.8307	-0.2186	1.0264
2	64.4901	-0.0224	1.7666
3	86.0189	-0.0077	2.5352
4	111.6051	-0.0023	2.1790
5	140.4797	0.0077	2.2562
6	164.0993	0.0207	1.7081

Table 1: Input values for the adjustment

With the values listed in Table 1, the collimation error, the tilting axis error and the eccentricity of the collimation axis can be computed according to Section 4.1 or 4.2. In this example, the computation is done as shown in Section 4.2 using a unit weight matrix (**P** = **E**) for the observations.

The collimation error is obtained as $\hat{c}$ = -37.80 mgon with $\sigma_{\hat{c}}$ = 5.36 mgon. For the tilting axis error we receive $\hat{i}$ = -30.17 mgon with $\sigma_{\hat{i}}$ = 1.90 mgon. The eccentricity of the collimation axis is computed as $\hat{e}$ = 1.17 mm with $\sigma_{\hat{e}}$ = 0.26 mm.

With the help of a *t*-test, we can test statistically if the computed values differ significantly from zero, see e.g. (Niemeier 2002, pp. 66 et seq.). Here, the test shows that (for a two-sided alternative hypothesis and a significance level of α = 5%) all values differ significantly from zero.

5. SELECTION OF OPTIMAL CONFIGURATION

Numerical investigations, based on the interpretation of the partial redundancies of the observations, have been used to find optimal measurement configurations. Using the approach of Section 4.2 we have to perform the following steps:

- Selection of tilt angles, which cover the measuring range from steep "upward" sights to steep "downward" sights, e.g. ζ_i = 10, 20..., 180, 190 gon.
- Estimation of the associated distances s_i to the target points.
- Performing an adjustment after Section 4.2 and additional computations of the partial redundancies (redundancy numbers) r_i, see e.g. (Niemeier 2002, p. 280), of the observations f_i.
- Successive elimination of the observation f_i with the largest partial redundancy (redundancy number), similar to the optimisation of observation plans in geodetic networks.

Numerical investigations have shown that the selection of eight target points represents a good compromise between economy and controllability of the observations. The minimum number of target points is six, like in the example of Section 4.3. The total redundancy (degrees of freedom) of the adjustment problem in this case is $r = 3$.

The question about the most favourable arrangement of the target points cannot be answered in general since the results dependent on the sighting distances. However, extensive numerical investigations have shown that the target points should be selected so that many steep "upward" and "downward" sights occur.

6. CORRECTION OF OBSERVATIONS

To correct for the axes errors and the eccentricity of the collimation axis, the Cartesian coordinates of the entire point cloud, received from a laser scan in Face 1, have to be converted into polar coordinates at first. After that the influence of the axis errors and the eccentricity on each direction can be computed with Equation (5). The desired spherical coordinates λ_i result from

$$\lambda_i = \alpha_{i_1} + f_i \quad , \tag{34}$$

and the desired spherical coordinates ϑ_i from

$$\cos\vartheta_i = \cos i \cos c \cos\zeta_{i_1} - \sin i \sin c \quad , \tag{35}$$

according to (Stahlberg 1997). The conversion into Cartesian coordinates follows thereafter.

Whether the observations have to be corrected for the computed axes errors and the eccentricity of the collimation axis depends on the required accuracy of a project. The position errors that are generated by these errors can be evaluated numerically. Using the errors obtained in Section 4.3, the respective error influences on the directions from Equation (5) are computed for tilt angles between 10 and 190 gon. Considering an average sighting distance of 15 m, the position deviations Δl_α (depicted on the left in Figure 8) result due to the erroneous directions. The influences of the axes errors on the tilt angles can be assessed by subtracting the corrected tilt angles ϑ_i, see Equation (35), from the "measured" tilt angles ζ_i. With sight distances of 15 m, the position deviations Δl_ζ (depicted on the right in Figure 8) result from erroneous tilt angles.

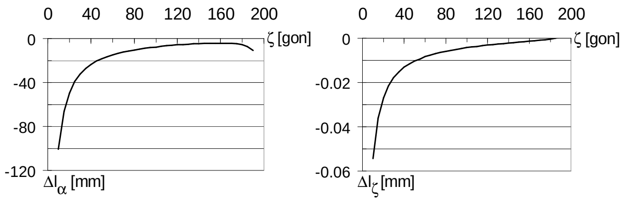

Figure 8: Position deviations Δl_α and Δl_ζ due to axes errors and eccentricity

In this example, the effect of the axes errors and the eccentricity of the collimation axis on the directions causes position deviations Δl_α of several centimetres for steep sights. The effect of the axes errors on the tilt angles is much smaller, giving position deviations Δl_ζ of less than a tenth of a millimetre even for very steep sights. Therefore, the tilt angles need not to be corrected for the axes errors in terrestrial laser scanning practice.

7. CONCLUSION

With the developed formulae, the user has access to a feasible method for the determination of the collimation error, the tilting axis error and the eccentricity of the collimation axis of a terrestrial laser scanner with a tacheometric measurement principle. It would be desirable if an input (into the instrument's software) of the axes errors and the eccentricity of the collimation axis were possible, so that the laser scanner would produce corrected coordinates of the point cloud so that no further computations were required by the user. Further investigations should examine whether reliable determinations of the instrumental errors are possible with simpler two-dimensional targets (e.g. printable targets).

REFERENCES

Deumlich, F. & Staiger, R. (2002): *Instrumentenkunde der Vermessungstechnik*. 9., völlig neu bearbeitete und erweiterte Auflage. Herbert Wichmann Verlag, Heidelberg.

Koch, K. R. (2000): *Einführung in die Bayes-Statistik*. Springer-Verlag, Berlin, Heidelberg, New York.

Niemeier, W. (2002): *Ausgleichungsrechnung*. Walter de Gruyter, Berlin, New York.

Stahlberg, C. (1997): Eine vektorielle Darstellung des Einflusses von Ziel- und Kippachsenfehler auf die Winkelmessung. *Zeitschrift für Vermessungswesen (ZfV)*, 122 (5), pp. 225-235.

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