COMPARISON OF FORMS - A POSSIBLE CRITERION IN GEOMETRICAL DEFORMATION ANALYSIS

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Abstract

In geometrical deformation analysis a set of points is compared in two (or more) epochs. In case that the geodetic datum is unknown some transformation (e.g. Helmert transformation) has to be performed. However, the residuals after such a transformation are not easy to interpret, because they are composed of three parts:

residual after transformation = real deformation + transformation defect + random error

The only solution is to classify the points into groups of stable and instable. In standard approaches this decision is done using metric criteria. An example is given which shows that one of the standard solutions (localization with "S-transformation") can lead to an incorrect result.

As an alternative to this, our proposal is to use a comparison of forms which can be effected using the correlation coefficient. After a maximum correlation adjustment (MCA) and the computation of the correlation coefficient between the sets of coordinates associated with the two epochs as a criterion for the similarity of forms, we can be able to find stable and instable points even in cases where solutions based on metric criteria fail.

1 Deformation analysis and its basic principle

It is well known that the deformation analysis deals in first place with:

- monitoring of tectonic movements,
- periodic control survey of objects.

Another interesting field is a transformation of heterogeneous geodetic networks into a global datum. In this case the residuals often have considerable amounts. Therefore the question rises whether it would be reasonable also in this case to localize the stable (in the sense of less deformed) points and to use only them in order to determine the transformation parameters.

Independently of which of these cases is considered we have always the same basic principle. On the basis of repeated measurements two (or more) epochs are compared.

2 Two essentially different cases

In practice we can divide the problem into two essentially different cases:

1. The geodetic datum is known in advance.
2. The geodetic datum is unknown (general case).

2.1 Known geodetic datum

In the case that the geodetic datum is known in advance the residuals are composed only of two parts:

computed deviation = deformation + measurement error

The decision if the deviations result from deformations is usually done by using statistical tests.

2.2 Unknown geodetic datum

In the case that the geodetic datum is unknown a transformation is needed in order to transform the epochs into a common datum. Therefore the transformation parameters have to be determined. This is usually done using a least squares adjustment (e.g. Helmert transformation in a plane), but the residuals after such a transformation are not easy to interpret (it is sometimes even impossible) because they are composed of three parts:

residual after transformation = real deformation + transformation defect + random error

The only solution is to classify the points into groups of stable and instable points but the obtained transformation parameters are not always suitable for this decision because of the transformation defect caused by using instable points.

For a successful determination of the transformation parameters it is important to use only stable points.

3 Comparison of two epochs

For a comparison of two epochs a lot of solutions can be found in the well-known literature. An overview of standard solutions is included in (WELSCH et. al. 2000).

All common solutions are based on analyzing amounts of residuals (e.g. by minimizing them by a least squares adjustment) so that they are all based on *metric criteria*.

Knowing that this procedure doesn’t function in every case (see section 4) it is logical to raise the question if there are other possible criteria.

4 An example

The following example (Fig. 1) is taken from (REINKING 1994). It is a trilateration network which was measured in two epochs so that it is possible to determine the coordinates in both of them (Table 1).

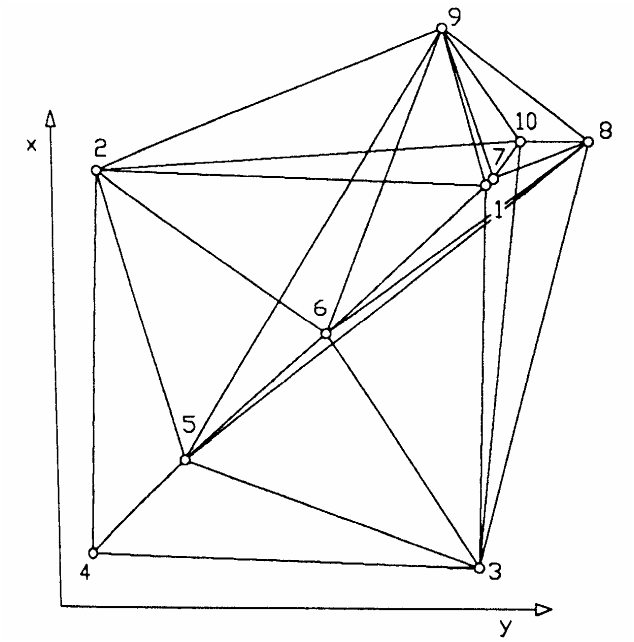

Figure 1: Geodetic network (trilateration)

Table 1: Coordinates of the network

	Epoch I		epoch II	
point	Y [m]	X [m]	y [m]	x [m]
1	220.00	220.00	222.00	217.50
2	20.00	220.00	22.50	222.50
3	220.00	20.00	217.50	17.50
4	20.00	20.00	16.00	25.50
5	70.00	70.00	68.00	73.00
6	140.00	140.00	140.00	140.50
7	225.00	220.00	225.00	220.00
8	275.00	240.00	275.00	240.00
9	200.00	300.00	200.00	300.00
10	240.00	240.00	242.00	237.50

The points 7, 8 and 9 are stable (identical coordinates) but this information will not be used in the calculation and the geodetic datum will be regarded as unknown.

4.1 Deformation analysis based on metric criteria

In the case that the geodetic datum is unknown some kind of transformation has to be applied. REINKING (1994) used a method called "localization with S-transformation" with a fixed scale between both epochs. The value of the test quantity R_f (some sort of a sum of squared deviations, for details see (NIEMEIER 1985)) was calculated and the point associated with the lowest value of R_f was assumed to be instable. In Fig. 2 it is shown that point 9 is associated with the lowest value so that this point should be eliminated from the group of stable points. However, we know that point 9 was a stable point and this method cannot lead to a correct result in this example.

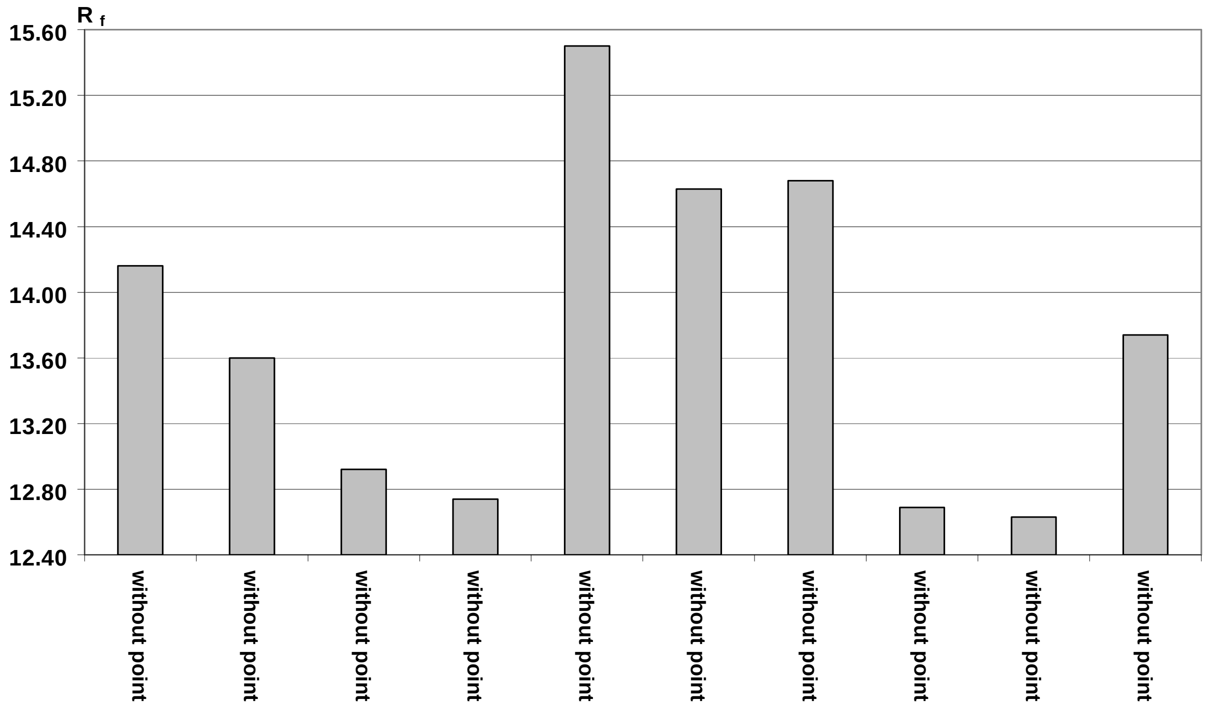

Figure 2: Value of R_f for each point using ”Localization with S-transformation”

Another possibility would be computing n Helmert transformations (scale not fixed), each with n-1 points. After that, we can see which combination results in the smallest value of mean error m_0. The point whose exclusion is associated with the lowest value of m_0 is assumed to be instable. In Fig. 3 we can see that again point 9 should be excluded from the candidates for stable points. So we get the same wrong result as before.

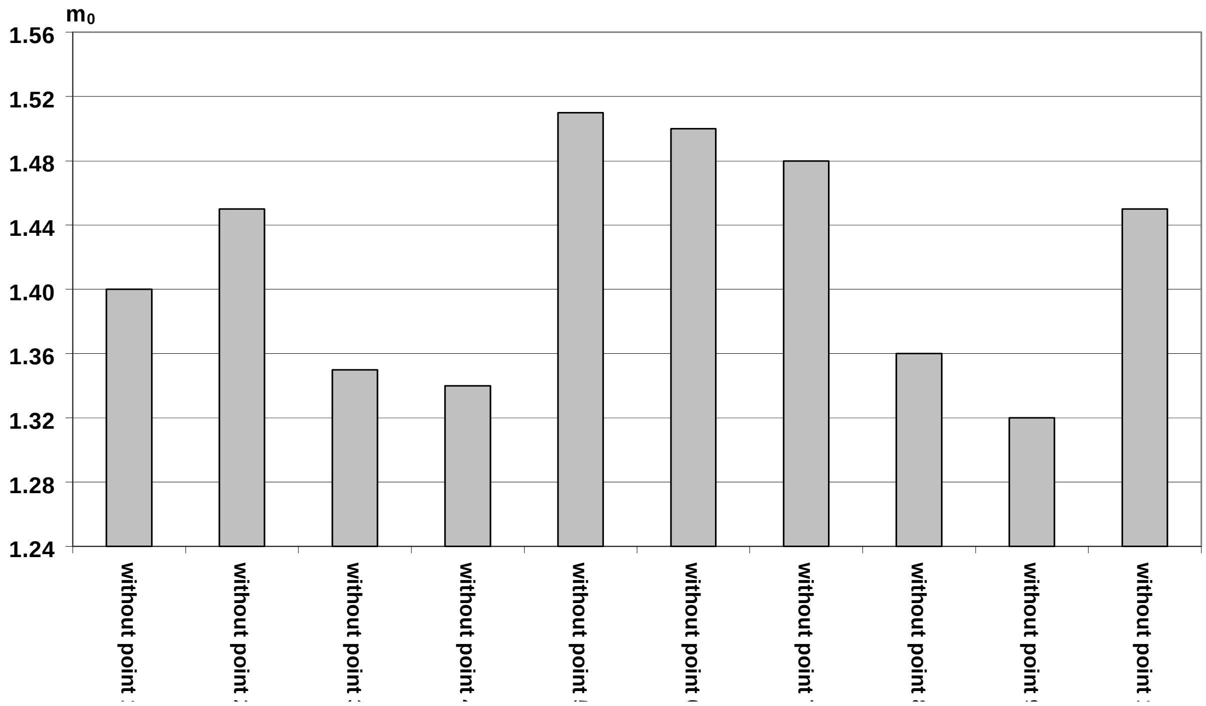

Figure 3: Value of m_0 for n Helmert transformations, each with n-1 points

5 Comparison of forms

5.1 Correlation coefficient

As an alternative to the analysis based on metric criteria our proposal is to use a *comparison of forms*. This comparison can be based on similarity which is described by the correlation coefficient (squared) r^2. The well-known definition for the correlation coefficient for two sets of real numbers (one-dimensional case) is

$$r^2 = \frac{\left(\sum_{i=1}^{n}(w_i - \overline{w})(z_i - \overline{z})\right)^2}{\sum_{i=1}^{n}(w_i - \overline{w})^2 \sum_{i=1}^{n}(z_i - \overline{z})^2}$$

with

$$w_i \in \quad, z_i \in \quad, (i = 1, \dots , n)$$

and

$$\overline{w} = \frac{1}{n}\sum_{i=1}^{n} w_i \,, \quad \overline{z} = \frac{1}{n}\sum_{i=1}^{n} z_i \quad .$$

The points of a geodetic network are not just real numbers, therefore we have to define the correlation coefficient in a generalized form. In order to compare two groups of points several ways are possible. For each epoch there are n quantities which can be regarded as vectors in a m-dimensional vector space (e.g. m = 2 in a plane):

$$\mathbf{w}_i = \left[w_{i_1}, w_{i_2}, \dots, w_{i_m}\right]^{\mathrm{T}} \in \quad {}^{m}, (i = 1, \dots , n)$$

and

$$\mathbf{z}_i = \left[z_{i_1}, z_{i_2}, \dots, z_{i_m}\right]^{\mathrm{T}} \in \quad {}^{m}, (i = 1, \dots , n)$$

containing the coordinates of points in epoch 1 resp. 2.

One possibility to define the correlation coefficient is to take the same formula as for real numbers but to replace the multiplication of real numbers with the inner product of vectors:

$$r^2 = \frac{\left(\sum_{i=1}^{n}(\mathbf{w}_i - \overline{\mathbf{w}})\cdot(\mathbf{z}_i - \overline{\mathbf{z}})\right)^2}{\sum_{i=1}^{n}(\mathbf{w}_i - \overline{\mathbf{w}})^2 \sum_{i=1}^{n}(\mathbf{z}_i - \overline{\mathbf{z}})^2}$$

with

$$\overline{\mathbf{w}} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{w}_i \quad \text{and} \quad \overline{\mathbf{z}} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{z}_i \quad .$$

Another possibility is putting all components of all points into a single vector so that we have in fact only two vectors

$$\mathbf{w}^{\mathrm{T}} = \left[\mathbf{w}_1{}^{\mathrm{T}}, \mathbf{w}_2{}^{\mathrm{T}}, \ldots, \mathbf{w}_n{}^{\mathrm{T}}\right] = \left[w_{1_1}, \ldots, w_{1_m}, w_{2_1}, \ldots, w_{2_m}, \ldots, w_{n_1}, \ldots, w_{n_m}\right]^{\mathrm{T}}$$

and

$$\mathbf{z}^{\mathrm{T}} = \left[\mathbf{z}_1{}^{\mathrm{T}}, \mathbf{z}_2{}^{\mathrm{T}}, \ldots, \mathbf{z}_n{}^{\mathrm{T}}\right] = \left[z_{1_1}, \ldots, z_{1_m}, z_{2_1}, \ldots, z_{2_m}, \ldots, z_{n_1}, \ldots, z_{n_m}\right]^{\mathrm{T}}$$

for a comparison of two epochs. In this case the correlation coefficient has the form

$$r^2 = \frac{\left((\mathbf{w}-\overline{\mathbf{w}}) \cdot (\mathbf{z}-\overline{\mathbf{z}})\right)^2}{(\mathbf{w}-\overline{\mathbf{w}})^2\ (\mathbf{z}-\overline{\mathbf{z}})^2}$$

with

$$\overline{\mathbf{w}} = \tfrac{1}{nm}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{w}_{i_j}, \ldots, \sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{w}_{i_j}\right]^{\mathrm{T}} \quad \text{and} \quad \overline{\mathbf{z}} = \tfrac{1}{nm}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{z}_{i_j}, \ldots, \sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{z}_{i_j}\right]^{\mathrm{T}} .$$

A further possibility could be to compute the one-dimensional correlation coefficient for each coordinate x, y (in a plane) taken alone and to define the correlation coefficient between two epochs as a product of the coefficients for each coordinate:

$$r^2 = r(x)^2 \cdot r(y)^2 \quad .$$

In the following investigations the correlation coefficient between n two-dimensional vectors is used (the first form).

5.2 Maximum correlation adjustment (MCA)

The amount of the correlation coefficient r^2 is a consequence of:

1. the extent of the similarity of forms,
2. the relative position of the configurations.

This is illustrated on a simple geometrical example with two similar triangles. In order to compute a correlation coefficient we have to use coordinates as it is visible from the definition in section 5.1. Hence we must choose some coordinate systems. In the relative position shown in Fig. 4 the result of the computed correlation coefficient is $r^2 = 1$.

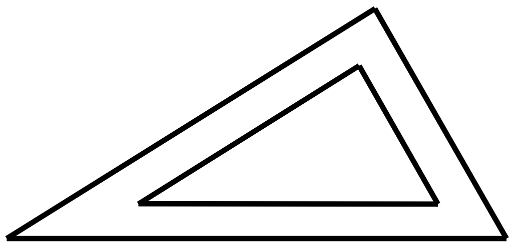

Figure 4: Two triangles in a homothetic position

If we choose the coordinate systems in such a way that we get the position of the triangles shown in Fig. 5 the correlation coefficient will no longer be $r^2 = 1$.

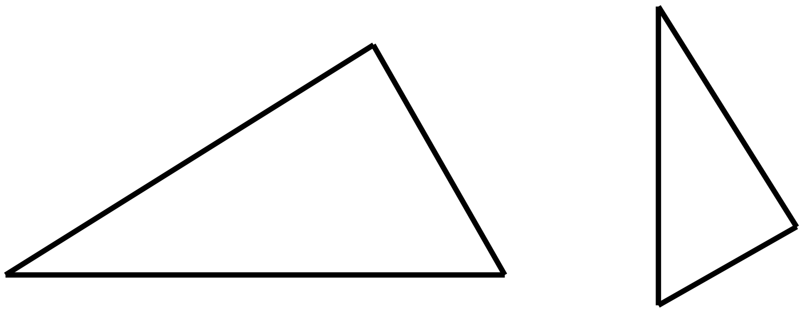

Figure 5: Two triangles in an arbitrary position

Therefore, in order to determine only the extent of similarity we must consider all possible relations of both coordinate systems and choose the position which yields the maximum correlation coefficient. This is done by a maximum correlation adjustment introduced in (PETROVIC 1991). The solution for a 4-parameter-transformation can be found in (NEITZEL 1999). In such a way we get the correlation coefficient r^2 which is *free from the influence of the coordinate systems* and it describes only the *extent of similarity*.

The solution of a maximum correlation adjustment is not unique. The result is a whole class of solutions, which includes the least squares solution, see (NEITZEL 1999). Inside this class we are able to find solutions that yield more realistic residuals, which allow us to interpret them in order to decompose the points into stable and instable ones.

Another possibility is to use the correlation coefficient for an analysis of parts of the network in order to find similar forms in both epochs. This possibility will not be discussed in detail in the present paper.

5.3 An example

As an example the same network from section 4 is used. We choose one solution from the class of all MCA-solutions and obtain the residuals v_x as shown in Fig. 6.

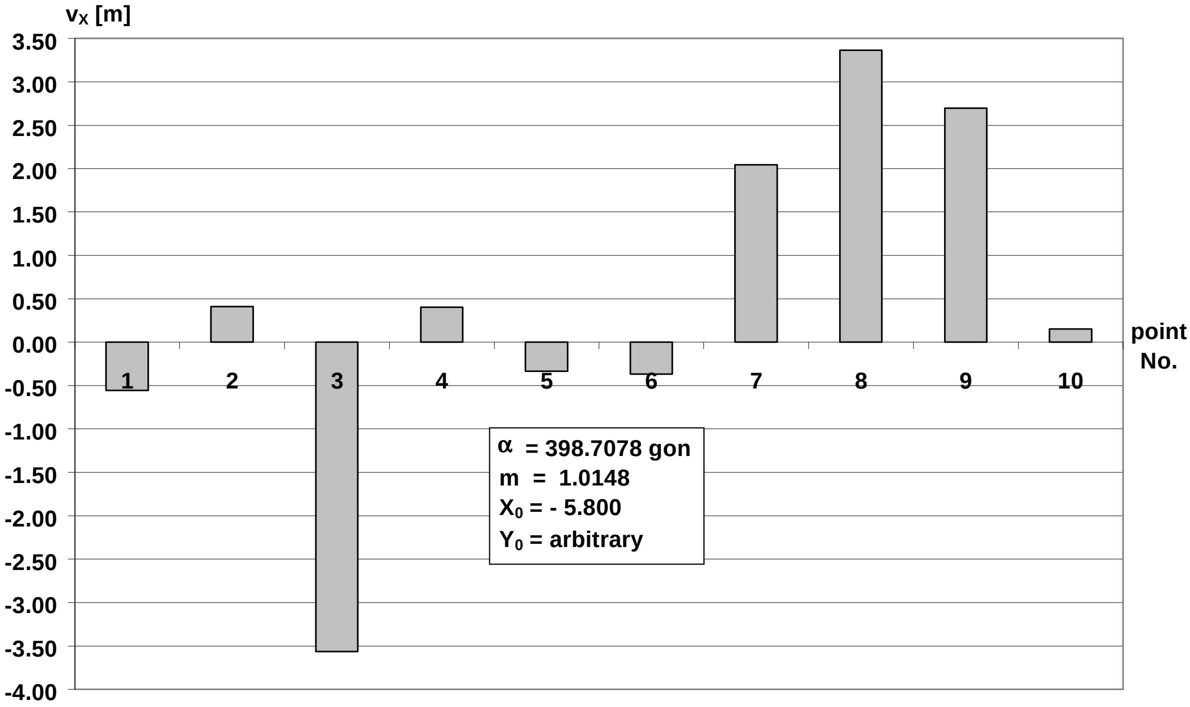

Figure 6: Residuals v_X for a chosen MCA-solution

Choosing another solution from the class of all MCA-solutions we obtain the residuals v_Y as shown in Fig. 7.

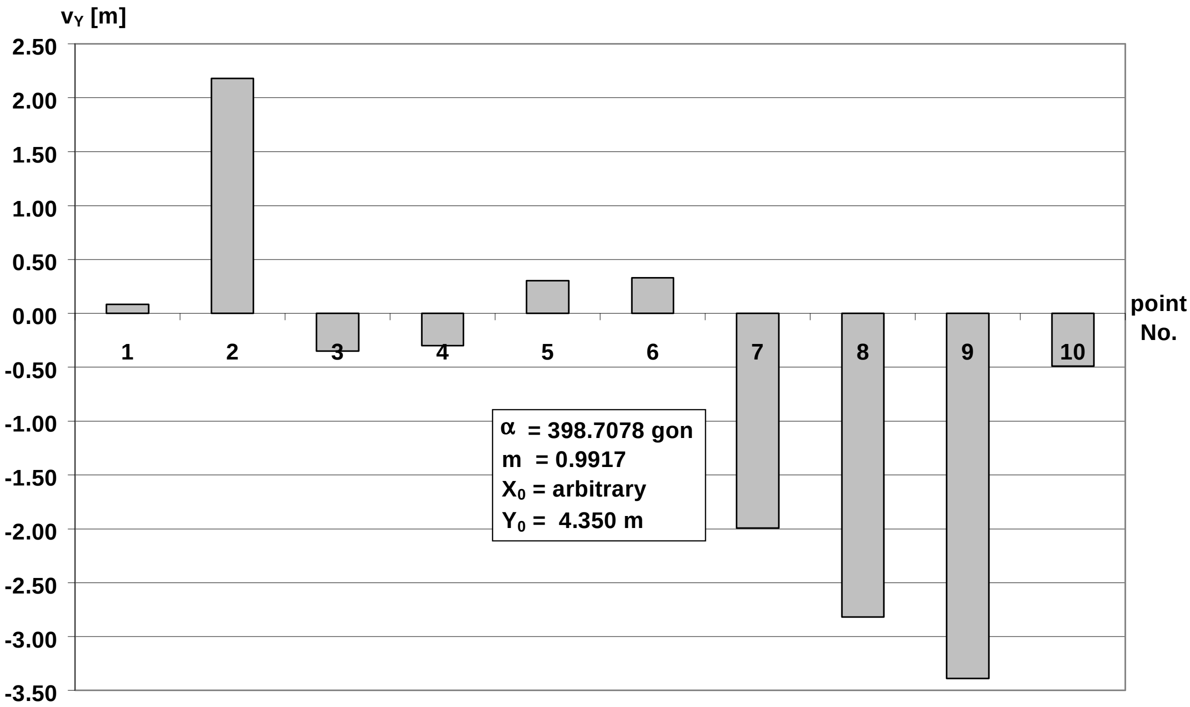

Figure 7: Residuals v_Y for a chosen MCA-solution

The interpretation of both sets of residuals is that the points can be decomposed into two groups:

- group 1 consisting of points 1, 2, 3, 4, 5, 6, 10,
- group 2 consisting of points 7, 8, 9.

After this decision the deformation analysis can continue. For example, a least squares adjustment for each group can be done. After this, we get the result that group 1 is instable and group 2 is the group of stable points.

6 Conclusion

In some cases an application of metric criteria results in a wrong identification of stable points. This is shown on a numerical example. A comparison of forms with a maximum correlation adjustment can make a reliable identification of groups with different deformations still possible. Using the correlation coefficient as a criterion offers two possibilities for this analysis:

1. Computation of more realistic residuals that allow us to interpret them in order to classify the points into stable and instable ones.
2. The use of the correlation coefficient for an analysis of parts of the network in order to find similar forms in both epochs.

The computation of more realistic residuals is demonstrated on the same numerical example on which metric criteria failed. Now it is possible to decompose the network into parts with different deformations.

7 References

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